MATHEMATICS
For Class XII

The question paper of Mathematics for Class XII will be based on the SLOs of the following unit:

I. FUNCTIONS AND LIMITS
II. DIFFERENTIATION
III. INTEGRATION
IV. INTRODUCTION TO ANALYTIC GEOMETRY
V. LINEAR INEQUALITIES AND LINEAR PROGRAMMING
VI. CONIC SECTIONS
VII. VECTORS

NATIONAL CURRICULUM

MATHEMATICS

CLASSES XI-XII

2000

GOVERNMENT OF PAKISTAN
MINISTRY OF EDUCATION
(CURRICULUM WING)
ISLAMABAD

National Curriculum

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PREFACE

In pursuance of National Education Policy (1998-2010), a project on Curriculum Reforms (Vision 2010) is in progress. It aims to improve the quality of education through curriculum revision and textbook development. The highest priority has been assigned to the revision of curriculum with a view to update the entire course contents so that Ideology of Pakistan could permeate the thinking of young generation and help them with necessary conviction and ability.

Believing in participatory approach the Ministry of Education requested the provincial governments/Curriculum Bureau to draft need based curricula in all the subjects for classes I through XII. Consequent upon this the Government of the Punjab attempted five initial drafts in Science and Mathematics. The Bureaus of Sindh, N.W.F.P. and Baluchistan furnished their comments on the previous as well as proposed curricula. To synchronize the feedback, the Ministry of Education appointed National Curriculum Development Committees. The panels of the committees were comprised of curriculum developers, subject specialists, educationists, teachers of universities, schools and colleges. The representatives of National Curriculum Bureau and Provincial Curriculum Bureaus were also represented on the panels. The committees thoroughly analyzed and synthesized the comments. Global experiences of curriculum development were also kept in view while revising/ updating the National Curriculum.

In the light of the above considerations, the committees revised the existing National Curriculum in Elementary Science (I-VIII), Physics, Chemistry, Biology (IX-XII), Statistics (XI, XII), Computer Science (IX-XII) and Mathematics (I-XII). The philosophy underlying National Curriculum is Islam and Ideology of Pakistan as set by the Parliament Act X, 1976. The objectives of the National Curriculum are framed in the light of the objectives of the latest National Education Policy (1998-2010). Purposeful learning competencies are suggested in each subject. These aim to provide the learners, skills for continuing education, civilized behaviour and attitude to become useful and peaceful citizens. The objective is also to provide them with the skills for economic development. The curriculum has been made more representative and responsive to the Ideology of Pakistan and social needs. We still believe that curriculum development is a continuous process and can be made more responsive. The Ministry would welcome comments from all concerned. This will help us in making the curriculum more effective and need based.

The Ministry of Education appreciates the contributions of all the Provincial Governments/ Curriculum Bureaux and the National Curriculum Development Committees.

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CONTENTS

Philosophy of Teaching Mathematics at the Higher Secondary School Level

Mathematics at the higher secondary school level is the gateway for entry not only to the field of higher Mathematics but also to the study of Physics, Engineering, Business and Economics. It provides logical basis of Set Theory, introduction to probability and problems of Trigonometry of oblique triangles. This is to be a standard course in Differential and Integral Calculus and Analytical Geometry which go a long way in making Mathematics as the most important subject in this age of science and technology.

Objectives Of Teaching Mathematics

Classes XI-XII

1 To provide the student with sound basis for studying Mathematics at higher stage.

2 To enable the student to apply Mathematics in Scientific and Technological fields.

3 To enable the student to apply mathematical concepts specifically in solving computational problems in Physics, Chemistry and Biology.

4 To enable the student to understand and use mathematical language easily and efficiently.

5 To enable the students to reason consistently, to draw correct conclusion from given hypotheses.

6 To inculcate in him the habit of examining any situation analytically.

Syllabus of Mathematics

For Class XI

Contents	Scope
Unit I Number Systems	
• Real Numbers Exercises	a. Review of the properties of real numbers as studied in secondary classes including the distinction between rational and irrational numbers. b. Proofs be given that the real numbers $\sqrt{2}$ and $\sqrt{3}$ are not rational numbers. c. π be introduced as an irrational number.
• Concept of Complex Numbers and Basic Operations on them. Conjugate and its properties. Modulus (absolute value) and its properties. Examples and Exercises.	• To know the solution of $x^2+1=0$ and i is as a symbol for $\sqrt{-1}$. Introduction to the concept of complex number as $x+iy$ and as an ordered pair (x,y) of real numbers. To Know the equality of complex numbers and to understand that there is no usual ordering ($<$ or $>$) property of complex numbers; to know four binary operations on complex numbers (distinct and repeated) and their properties (commutative, associative and distributive) ; to know the conjugate and modulus of a complex number $z=x+iy$; to know additive and multiplicative identities of complex numbers and to find their additive and multiplicative inverses. To know the proofs of the following: $\lvert -z\rvert = \lvert z\rvert = \lvert\bar{z}\rvert = \lvert -\bar{z}\rvert$ $\bar{\bar{z}} = z, \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2,$ $\overline{z_1 z_2} = \bar{z}_1 . \bar{z}_2, \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}, z_2 \neq 0,$ $z.\bar{z} = \lvert z\rvert^2$ Finding the real imaginary parts of $(x+iy)^n, \left(\frac{x_1+iy_1}{x_2+iy_2}\right)^n, x_2+iy_2 \neq 0$ where $n = \pm 1, \pm 2, \pm 3$
• Geometrical Representation of Complex numbers by Argand's Diagram. Examples and Exercises.	• To establish one to one correspondence between RxR and C (the set of all complex numbers); to know geometrical representation of complex numbers, their sum and difference in the plane by Argand's diagram, to know that for complex

	number $x + iy$, $x = r\cos\theta$, $y = r\sin\theta$ where r is modulus and θ is called argument; to establish the properties of complex numbers.
Unit II Sets, Functions and Groups	
• Revision of the work done in the previous classes. Examples and Exercise	• Sets and their types: operations on sets and verification properties of operations on the sets.
• *Logical Proofs of the Operation on Sets* Examples and Exercises	• Introduction to the logical statements (simple and compound) and their composition (common connectives, negation, conjunction, disjunction, conditional and biconditional); truth values and truth tables of logical statements and their logical equivalence, tautologies, contradictions and contigencies, quantifiers (universal and existential, analogy between the composition of logical statements and algebra of stes; to give formal proofs of the commutative, associative and distributive properties of union and *intersection and of the De-Morgan's laws;* illustration of the above mentioned properties of the operations on sets by Venn diagram.
• Functions. Examples and Exercises	• Definitions as a rule of correspondence, simple examples (Linear and quadratic functions, square root function),domain,codomain and range , one to one and onto functions and inverse functions.
• Binary Operations and its Different Properties Examples and Exercises	• To have the concept of a binary operation on aset and the idea of algebraic system; to know the properties of binary operations (closure, commutative, non-commutative, associative, non-associative, existence of identity and inverse in an algebraic system with respect to a given binary operation) Addition modulo and multiplication modulo be introduced,
• Groups Examples and Exercises	• To have the idea of an algebraic structure and to know the definitions of a groups ,a semi group, a monoid and a group ,finite and infinite groups; commutative (abelian)groups;non-*commutative(non-abelian) groups; solution of* equation $a * x = b$ $x * a = b$ in a group; a) cancellation laws $y*x = y*z \Rightarrow x = z$ $y*x = y*z \Rightarrow x = z$

	b) $(x^{-1})^{-1} = x$ and $(xy)^{-1} = y^{-1}x^{-1}$ c) to prove the uniqueness of the identity element and inverse of each element of a group
UNIT III Matrices and Determinants	
• Revision of the work done in the previous classes Examples and Exercises	• A matrix, its rows and columns and order of a matrix; transpose of a matrix, kinds of matrices(rectangular, square ,transpose,diagonal,scalar,null,unit matrix),equality of matrices' conformability of addition and multiplication of matrices' determinant of a 2 x 2 matrix, singular and non-singular, adjoint and inverse of a 2 x 2 matrix, and solution of simultaneous linear equations by using matrices.
• Operations on matrices Example and Exercises	• To have informal concept of a field and of a matrix of order m x n with entries from the field R of real numbers; to perform operations on 3 X 3 and simple cases on 4 x 4 matrices (In the case of matrices with entries as complex numbers, matrices of order2x2 only may be taken)to know the properties of the operations on matrices and to find that, in general ,multiplication of two matrices is not commutative
• Determinants and their Application in the study of the Algebra of Matrices Examples and Exercises	• Concept of a determinant of a square matrix expansion of the determinants up to order4(simple cases), to write minors and cofactors of the elements of a matrix; to find whether a matrix of order 3x3 and 4x4 is singular or non-singular, the properties of the determinants, the adjoint and the inverse of a matrix of order up to 3 x 3 and verification of $(AB)^{-1} = B^{-1}.A^{-1}$, $(AB)' = B'.A'$
• Types of Matrices and the Row and Column Operations on Matrices Examples and Exercises	• To know the elementary row and column operations on a matrix, To define the following types of matrices: Upper and lower triangular, symmetric and skew-symmetric, Hermitain and skew-hermitian and echelon and reduced echelon forms; to reduce a matrix to its echelon or reduced echolon form and be able to apply them in finding the inverse and rank (rank of matrix to be taken as number of non zero rows of the matrix in echelon form) of a matrix upto order 3 x 3
• Solving Simultaneous Linear System of Equations.	• To distinguish between systems of homogeneous and non-homogeneous linear equations in 2 and 3

Examples and exercises	unknowns, to know the condition under which a system of linear equation is consistent or inconsistent, to be able to solve a system of linear non-homogenous equations by the use of a) matrices i.e. $AX = B$, $X = A^{-1} B$ b) echelon and reduced echelon form • Cramer's rule
UNIT IV Quadratic Equations	
• Revision of the work done in previous classes. Exercises	• Solving a quadratic equation in on variable by: a. Factorisation b. completing the square and c. the quadratic formula
• Solution of Equations Reducible to Quadratic Equations is one Variable Examples and Exercises	• To solve equations reducible to quadratic equations in one variable (in the case of equations involving radicals ,the answers should be checked by substitution in the given equations so as to reject extraneous roots if any).
• Cube roots and Fourth Roots of unity Examples and Exercises	• to find cube roots and fourth roots of unity and their properties ω and ω^2 to be introduced as complex cube roots of unity
• Application of Remainder Theorem in the Solution of Equations Examples and Exercises	• To apply remainder theorem in finding one or two rational roots of cubic and quadratic equations, to use synthetic division in finding depressed equations for solving them.
• Relations between the Roots and the Co-efficient of Quadratic Equations. Examples and Exercises	• To establish the relations between the roots and co-efficient of a quadratic equation and their applications to find the nature of the roots of a quadratic equation with rational co-efficients.
• Formation of Quadratic Equations from given conditions Examples and Exercises	• To form quadratic equations in one variable whose roots are related in various ways with the roots of given quadratic equation
• Solution of a System of two Equations Examples and Exercises	• To solve a system of two equations, when a) one of them is linear and the other is quadratic in two variables b) both are quadratic equations in two variables .
• Problems on Quadratic Equations Examples and Exercises	• To solve word problems leading to quadratic equations
UNIT V Partial Fractions	
	• To define proper an improper rational fraction, to distinguish identities from conditional equations, to reduced a fraction into partial fractions when its denominate consists of a) linear factors

	b) Repeated linear factors (at the most cubes) • non-repeated quadratic factors.
UNIT VI Sequence and Series	
• Introduction Examples and Exercises	• To have the concept of a sequence/ progression, its term and its domain, different types of sequences with examples to distinguish between arithmetic, geometric and harmonic sequences; and to determine the sequence when its *nth* term is known
• Arithmetic sequence Examples and Exercises	• To find the *nth* term of an arithmetic progression(A.P.) and solve problems pertaining to the terms of an A.P.
• Arithmetic Mean/Means Examples and Exercises	• To have the concept of an arithmetic mean (A.M.) and of n arithmetic means between two numbers : to be able to find an A.M and to insert *n* A.Ms between two numbers.
• Arithmetic Series Examples and Exercise	• To know the definition of a series and its distinction from a sequence: to establish the formula for finding the sum upto *n* terms of an arithmetic series and be able to apply this formula
• Word problems on A.P Examples and Exercises	• To be able to solve word problems involving A.P
• Geometric Progression (G.P) • Examples and Exercises	• To be define geometric sequence, derive and apply the formula for its *nth* term
• Geometric Mean/Means Examples and Exercises.	• To find the geometric mean (G.M) and insert n G.Ms between two positive real members and be able to solve problems based on them.
• Geometric Series Examples and Exercises	• To establish the formulas for finding the sum of geometric series upto *n* terms and to infinity, and be able to apply them in finding the sum of the geometric series and evaluating recurring decimal fractions.
• Word Problems on G.P Examples and Exercise.	• To solve word problems leading to geometric progression/series.
• Harmonic sequence Examples and Exercises.	• To find the nth term of a harmonic progression (H.P) and apply it in solving related problems.
• Harmonic Mean and Means Examples and Exercises.	• Definition of harmonic mean and to insert n harmonic means between two numbers and solve problems on them, to prove that. a). $A > G > H$

	b). $G^2 = AH$ • where A, G, H have their usual meaning and $G>0$.
• Sum of the first *n* Natural Numbers, their Squares and Cubes. Examples and Exercises.	• To know the meaning of the symbol $\sum$, find the values of $\sum n$, $\sum n^2$, $\sum n^3$ and apply them in evaluating the sum of series.
Unit VII Permutations, Combinations and Probability	
• Factorial of a Natural Number Examples and Exercises.	• To know the meaning of factorial of a natural number and its notations and that $0! = 1$; to express the product of a few consecutive natural numbers in the form of factorials.
• Permutations Examples and Exercises.	• To know the fundamental principle of counting and to illustrate this principle using tree diagrams; to understand the meaning of permutations of n different things taken r at a time and know the notation ${}^nP_r$ or $P(n,r)$; to establish the formula for ${}^nP_r$ and apply it in solving problems of finding the number of arrangements of n things taken r at a time (when all the n things are different and when some of them are alike) and the arrangements of different things around a circle.
• Combinations Examples and Exercises.	• To know the definition of combinations of n different things taken r at a time, establish the formula for ${}^nc_r$ or $\binom{n}{r}$ or $C(n,r)$ and prove that ${}^nc_r = {}^nc_{n-r}$, ${}^nc_r + {}^nc_{r-1} = {}^{n+1}C_r$ • to apply combination in solving problems.
• Probability (Basic Concepts and Estimation of Probability). Examples and Exercises.	• To understand concepts of sample space, event and chances of its occurrence, equally likely events, mutually exclusive, disjoint, dependent, independent, simple and compound events the favourable chances for the occurrence of an event and probability; to know the formula for finding the probability; to apply the formula for finding probability in simple cases; to use Venn diagrams in finding the probability for the occurrence of an event.
• Addition and Multiplication of Probability Examples and Exercises.	• Know the following rules $P(S)=1$, $P(\varnothing)=0$, $0\le P(E)\le 1$ $P(E)=n(E)/n(S)$

	a. If X and Y are not compliments of each other, then $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ b. If X and Y are mutually exclusive and $(X \cup N) \subseteq S$, then $P(X \cup Y) = P(X) + P(Y)$ c. $P(X \cap Y) = P(Y).\ P(X/Y)$, where $P(X/Y)$ is conditional probability of X when Y has already occurred, $P(Y) \neq 0$ d. If X and Y are independent events, then $P(X \cap Y) = P(Y).\ P(X)$. e To be able to solve problems related to the above stated rules of addition and multiplication of probability.
Unit VIII Mathematical Induction and Binomial Theorem	
• Introduction and Application of Mathematical Induction. Examples and Exercises.	• To know the principle of mathematical induction and its various applications.
• Binomial Theorem for Positive Integral Index. Examples and Exercises.	• To state and prove the binomial theorem for positive integral index, find the number of terms and general term in the expansion of $(a + b)^n$ and apply it to expand positive integral powers of the binomials and find their particular terms (without expansion).
• Binomial Theorem for Negative Integral and Rational Indices. Examples and Exercises.	• To state binomial theorem for negative integral and rational indices and find its general term; to be apply the theorem in the expansion of the binomial expressions with rational indices as infinite series and arithmetical computations.
• Binomial Series Examples and Exercises.	• To be able to identify given series as a binomial expansion and hence find the sum of the series.
Unit IX Fundamentals of Trigonometry	
• Introduction	• To know the meaning and importance of trigonometry in the study of higher mathematics and other branches of knowledge which will be a source of motivation of its study.
• Units of Measures of Angles. Examples and Exercises.	• To know the sexagesimal system of measure of an angle and the mutual conversion of the units of sexagesimal system. To know the definition of a radian as unit of the measurement of angles in the circular system and be able to convert the measures of one system to another; to have the concept of the measure of an angle as the amount

	of rotation including the senses of clock wise and anti-clock wise rotation so as to have the idea of general angle and circular residue.
• Relation between the Length of an arc of a circle and the circular measure of its central angle. Examples and Exercises.	• To establish the rule $\theta = l/r$, where r is the radius of the circle, l is the length of the arc and θ is the circular measure of the central angle of the arc.
• Trigonometric Functions Examples and Exercises.	• To know the definitions of the six basic trigonometric ratios of an angle; to be able to find the values of the trigonometric ratios of the angles of the measures upto 90^0 by using tables/calculator; to know the signs of the trigonometric ratios of angles with their terminal arms in the four quadrants; to know the value of basic trigonometric ratios of the angles of the following measures; 0^0, 30^0, 45^0, 60^0, 90^0, 180^0, 270^0, 360^0; to establish the following relations between the trigonometric ratios; $\text{cosec}\,\theta = 1/\sin\theta$, $\sec\theta = 1/\cos\theta$, $\cot\theta = 1/\tan\theta$, $\tan\theta = \sin\theta/\cos\theta$, $\cot\theta = \cos\theta/\sin\theta$, $\sin^2\theta + \cos^2\theta = 1$, $1+\tan^2\theta = \sec^2\theta$ and $1+\cot^2\theta = \text{cosec}^2\theta$: to be able to apply the above mentioned relations in (a) finding the values of 5 basic ratios in terms of the 6th ratio and **(b)** proving the trigonometric identities; **(c)** to have the concepts of radian function, trigonometric functions, domain and range of trigonometric functions.
Unit-X Trigonometric Identities of Sum and Difference of Angles	
• Fundamental Formulas of Sum and Difference of two Angles and their Application. Examples and Exercises	• a) Informal introduction of distance formula; b) to establish the formula: $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ and deductions there from for finding the sum and difference of the trigonometric ratios: to be able to apply them.
• Trigonometric Ratios of Allied Angles Examples and Exercises	• To find the trigonometric functions of angles of radian measures, $\theta, \pi\pm\theta$. $\pi/2\pm\theta$, $3\pi/2\pm\theta$, $2\pi\pm\theta$ and apply them.
• Trigonometric Ratios of Double Angles and Half Angles	• To find the values of the trigonometric ratios of double and half the angles and apply them.

Examples and Exercises	
• Sum, Difference and Product of the Trigonometric Ratios.	• To find the formulas for the following: $\sin\alpha \pm \sin\beta$; $\cos\alpha \pm \cos\beta$; $2\sin\alpha\cos\beta$; $2\cos\alpha\sin\beta$; $2\sin\alpha\sin\beta$ and $2\cos\alpha\cos\beta$ and to be able to apply them.
UNIT XI Trigonometric Functions and their Graphs	
• Periods of Trigonometric Functions Examples and Exercises	• To know the domains and ranges of the trigonometric functions to have the concept of period of a trigonometric function and the periods of the basic trigonometric functions.
• Graphs of Trigonometric Functions Examples and Exercises	• To draw the graphs of the six basic trigonometric functions in the domains ranging from -2π to 2π and know that the graphs of these trigonometric functions are repeated depending upon the periods of the functions.
UNIT XII Application of Trigonometry	
• Heights and Distances Examples and Exercises	• To be able to use solution of right triangles in solving the problems of heights and distances.
• Cosine Formula Examples and Exercises	• To establish the cosine formula and apply it in the solution of oblique triangles.
• Sine Formula Examples and Exercises	• To establish the sine formula to apply it in the solution of oblique triangles.
• Tangent Formula Examples and Exercises	• To establish the tangent formula and to apply it in the solution of oblique triangles.
• Values of the Trigonometric Ratios of Double and Half angles of a Triangles in terms of the sides. Examples and Exercises	• To derive the formula of double and half angles of a triangle by using the law of cosine and apply it.
• Areas of Triangular Regions. Examples and Exercises	• To establish and apply the following formulas for finding the areas of triangular regions a. $1/2\, ab \sin\gamma$, $1/2\, bc \sin\alpha$, $1/2\, ca \sin\beta$ b. $\frac{1}{2} a^2 \sin\beta \sin\gamma/\sin\alpha$ $\frac{1}{2} b^2 \sin\alpha \sin\gamma/\sin\beta$ $\frac{1}{2} c^2 \sin\alpha \sin\beta/\sin\gamma$ c. $\sqrt{s(s-a)(s-b)(s-c)}$
• Radii of Circles connected with Triangles. Examples and Exercises	• To find the radii of a. Circum circle b. In circle c. Escribed circle of

	triangles and to solve problems involving these radii.
UNIT XII **Inverse Trigonometric Functions**	
• Inverse Trigonometric Functions Examples and Exercises	• To know the definition of inverse trigonometric functions their domains and ranges; to know the general and principal trigonometric functions their inverses and their values; development of formulas for inverse trigonometric functions and their applicants; to draw the graphs of inverse trigonometric functions.
UNIT-XIV **Solutions of Trigonometric Equations**	
• Solution of Trigonometric Functions Examples and Exercises	• To solve trigonometric equations and check their answers by substitution in the given equations so as to discard extraneous roots and to make use of the periods of trigonometric functions for finding the general solutions of the equations.

Syllabus of Mathematics for Class XII

Contents	Scope
Unit 1 Functions and Limits	
• Revision of the work done in the previous classes. Exercises	• Function, its domain and rage; series (geometric series and binomial series); graphs of algebraic linear functions, trigonometric functions and inverse trigonometric functions.
• Kinds of Functions Examples and Exercises	• To know the following types of functions: algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic, inverse hyperbolic; explicitly and implicitly defined functions, parametric representation of functions, even and odd functions.
• Composition and Inversion of Functions. Examples and Exercises	• To know the meaning of the identity and constant functions and the techniques of composing and inverting the functions by algebraic methods.
• Limits of Functions and Theorems on limits. Examples and Exercises	a) explanation of the terms: $x \to a$ and $x \to 0$ and $x \to \infty$ b) intuitive notion of limit of a function at a point and at ∞, illustration with suitable examples. c) theorems on sum, difference, product and quotient of function.
d. Limits of important functions Examples and Exercises	e. a) limit of the following functions at $x=a$ $\frac{(x^n-a^n)}{(x-a)}, \frac{(x-a)}{(\sqrt{x}-\sqrt{a})}$ b) limit of $\left(1+\frac{1}{x}\right)^x$ at ∞ c) limits of the following functions at $x=0$ $\frac{\sqrt{x+a}-\sqrt{a}}{a}; (1+x)^{1+x}; \left(\frac{a^x-1}{x}\right)$ $\frac{\sin x}{x}$ and their application in evaluation of the limits of algebraic, exponential and trigonometric functions.
• Continuous and Discontinuous Functions Examples and Exercises	• To understand the concept of continuity of a function at a point and in an interval intuitively,

	explanation of continuity and discontinuity through graphs.
• Graphs Examples and Exercises	• To draw the graphs of a) Explicitly defined functions like $y=f(x)$, where $f(x)=e^x$, a^x, $\log_a x$, $\log_e x$ b) Implicitly defined functions such as $x^2 + y^2 = a^2$; $x^2/a^2 + y^2/b^2 = 1$ distinction between graph of a function and graph of an equation must be stressed. c) Parametric equations of functions such as $x=at^2$, $y=2at$; $x = a\sec\theta$, $y = b\tan\theta$ d) Discontinuous functions of the type $y=\begin{cases} x \text{ when } 0 \le x \le \text{ and } \frac{x-4}{2}, x \ne 2 \\ x-1 \text{ when } 1 \le x \le 2 \end{cases}$ and solve the following equations graphically: $\cos x = x$; $\sin x = x$; $\tan x = x$;
Unit II Differentiation	
• Introduction Example and Exercises.	• Concept of dependent and independent variables, average rate of change of a variable w.r.t another variable, instantaneous rate of change of variable w.r.t another variable, definition of derivative (differential coefficient)
• Differentiation of Algebraic Expressions Exeample and Exercises.	• Calculation of derivatives from definition, average rate of change. • Instantaneous rate of change. • To be able to calculate the derivatives of $y = (ax+b)^n$, $n = 1,2,3,$.......... $y = \dfrac{1}{(ax+b)^n}$, $n = 1,2,3,$........ by definition (ab-initio)
• Theorems of Differentiation Example and Exercises.	• To established of the theorems on differentiation sum, difference, product and quotients of functions and their application, differentials of $y = (ax + b)^n$ *where n is negative integer*, using quotient theorem.
• Chain Rule Example and Exercises.	• Explanation and application of chain rule for composite function and functions defined by parametric equations.
• Differentiation of Functions other than algebraic. Example and Exercises.	• To find the derivatives of trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic functions using

	chain and other rules. Derivation of $y = x^n$ where $n = p/q$, $q \neq 0$.
• Successive differentiation. Example and Exercises.	• To have the concept of successive differentiation. • To find 2nd, 3rd and 4th derivatives of algebraic, trigonometric, exponential and logarithmic functions. • To find 2nd derivatives of implicit, inverse trigonometric and parametric functions defined by parametric equations.
• Maclaurin's and Taylor's Theorems. Example and Exercises.	• To know the Maclaurin's and Taylor's theorems with application in simple cases only.
• Extreme Values. Example and Exercises.	• To know the geometrical interpretation of the derivative of a function, (as a slope of the tangent line at a point to the graph of $y = f(x)$). • To find whether a function is increasing or decreasing at a point and in an interval. • To have the concept of turning point (extreme point) • To have the concepts of maximum and minimum values and critical points of a function. • To know the second derivative test of maxima and minima. • To solve simple word problems of maxima and minima.
Unit III Integration	
• Differentials Example and Exercises.	• To have the concept of differentials and to a. distinguish between dy and δy, b. find *dy/dx* using differentials c. simple application of differentials in finding approximate values of irrational numbers and sin x, cos x, when x = 29°, 46°, 62°, etc.
• Introduction to Integration. Example and Exercises.	• To define integration as anti-derivative (inverse of derivative) and to know simple standard integrals which directly follow from standard differentiation formulas and to apply them in the integration of functions of the same types.
• Theorems on Anti-derivatives Example and Exercises.	• To know the theorems (without proof) on antiderivatives of a. constant multiple of a function

	b. sum and difference of functions and their applications.
• Integration by Substitution Examples and Exercises	• To know and be able to integrate by applying the method of substitution in the integration of functions including the following standard forms: $\frac{1}{\sqrt{x^2-a^2}}$, $\frac{1}{\sqrt{x^2+a^2}}$
• Integration by Substitution Examples and Exercises	• To know and be able to find the antiderivtives of functions by parts including the following standard forms. $\sqrt{a^2-x^2}$, $\sqrt{x^2+a^2}$, $\sqrt{x^2-a^2}$
• Integration involving Partial Fractions Examples and Exercises	• To be able to use partial fractions in integration of rational functions having denominators consisting of: a. linear factors b. repeated linear factors (up to 3) c. linear and non-repeated quadratic factors.
• Definite Integrals Examples and Exercises	• To be able to differentiate between definite and indefinite integrals and to know and apply the following theorems of definite integrals. a. definite integral: $\int_a^b f(x)dx$ as the area under the curve y = f(x) from x = a to x = b and the x-axis b. Fundamental theorems of calculus c. $\int_a^b f(x)dx = -\int_a^b f(x)dx$; $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^a f(x)dx$
• Application of definite Integrals Examples and Exercises	• To be able to calculate areas bounded by the curve and x-axis.
• Differential Equations Examples and Exercises	• To have the concept of a differential equation and its order. • To be able to solve differential equations of first order with variables separable in the forms; dy/dx=f(x)/g(y) or dy/dx = g(y)/f(x) concept of initial conditions and simple applications.

Unit IV **Introduction to Analytic Geometry**	
Coordinate System Examples and Exercises	• Be able To: a. locate a point in a Cartesian Plane, b. derive the distance formula, c. divide the line segment in given ratio (internally and externally), find the mid-point of a line segment. d. Apply the above results in proving concurrency of the right bisectors, bisectors of the angles, medians and altitudes of a triangle.
• Translation and Rotation of Axes Examples and Exercises.	• Be able to find the coordinates of a point under transition and rotation of axes.
• Equations of straight lines. Examples and Exercises.	a. Concept of the slope of a line. b. To find the slope of a line passing through two points; the equations of the x-axis and y-axis; c. The equations of the straight lines parallel and perpendicular to the coordinate axes. d. Derivation of the following standard forms of the equations of straight lines: shope-intercept; point-slope ; two points; intercepts; normal and symmetric. e. To establish the fact that a linear equation in two variables x and y represents a straight line. f. To transform the linear equation $ax + by + c = 0$ in the standard forms listed in (d) above. g. To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines. h. To find the area of a triangle whose vertices are given.
i. Two and Three Straight lines. Examples and Exercises.	Be able to find: a. the point of intersection of two straight lines. b. the condition of concurrency of three straight lines and their point of concurrency. c. acute angle between two straight lines, condition of their parallelism and perpendicularty. d. the equation of lines through the point of intersection of two lines with a given condition (including parallelism and perpendicularity). e. the equation of the right bisector of a line segment, f. the equations of the medians, altitude and right

	bisectors of a triangle when its vertices or equations of sides are given. g. area of triangle when equations of its sides are given. h. Equations of one, two or three straight line/s and the condition of concurrency of three straight lines in matrix form.
• Homogeneous Equations of 2^{nd} Degree in two Variables x and y. Examples and Exercises.	• Concept of homogeneous equations in one or two variables. • To show that a 2^{nd} degree homogeneous equation in two variables x and y represents a pair of straight lines through the origin. • To find the angle between these lines. • To find the condition of coincidence and perpendicularity of these lines and their applications.
Unit V Linear Inequalities and Linear Programming	
• Linear Inequalities and their Graphs Examples and Exercises.	• To know the meaning of linear inequalities in two variables and their solutions be graphically illustrated; determine graphically the region bounded by 2 or 3 simultaneous inequalities of non negative variables and shading the regions bounded by them.
• Feasible Solution Set Examples and Exercises.	• To know feasible solution set and graphically find the feasible solution sets of the problems from every day life.
• Linear Programming Examples and Exercises.	• To have the concepts of simple linear programming and of optional solution of the linear objective functions and to find the optional solution of the linear objective functions by graphical methods.
UNIT VI Conic Sections	
• Introduction	• To know that circle, parabola, ellipse and hyperbola are sections of cones.
• Circle Examples and Exercises.	a. To know the definition of a circle b. to derive the equation of circle in the form $(x-h)^2 + (y-k)^2 = r^2$ c. to know the general form of equation of circle as $x^2 + y^2 + 2gy + 2fy + c=0$ and be able to find its centre and radius. d. to find the equation of a circle:

	1. Passing through three non collinear points 2. Passing through two points and having its centre on a given line 3. Passing through two points and equation of tangent at one of these points its known. 4. Passing through two points and touching a given line.
• Tangents and Normals Examples and Exercises.	• To find: a. the points of intersection of a circle with a line including the condition of tangency. b. the equation of a tangent to a circle in slope form. c. the equations of tangent and a normal to a circle. d. at a point e. when its parametric equations are given f. when the length of tangent to a circle from an external point is given g. to prove that two tangents drawn to a circle from an external point are equal in length.
• Analytic proofs of important properties of a circle. Examples and Exercises.	• To prove analytically the following properties of a circle: a. the perpendicular from the centre to a chord bisects it and its two converses. b. the congruent chords of a circle are equidistant from its centre and its converse. c. the measure of the central angle of a minor arc is double the measure of the angle subtended in the corresponding major arc. d. the angle in a semi-circle is a right angle and its converse. e. the perpendicular at the outer end of a radial segment is tangent to the circle and its two converses.
PARABOLA	
• Parabola and its elements Examples and Exercises.	• to know the concept of a parabola and its elements (focus, directrix, eccentricity, vertex, axis, focal chord, latus rectum) • to derive the standard forms of equations of parabolas and to draw their graphs and to find the elements.

• Equation of a Parabola with given elements Examples and Exercises.	• a) to find the equation of a parabola with the following given elements – focus and vertex – focus and directrix – vertex and directrix
• Tanagents and Normals to a Parabola Examples and Exercises.	• To find a) points of intersection of a parabola with a line including the condition of tangency. b) the equation of a tangent in slope form, c) the equation of a tangent and a normal to a parabola at a point • Applications, suspension and reflection properties of parabola.
Ellipse	
• Ellipse and its elements Examples and Exercises.	• To know the concept of an ellipse and its elements (centre, foci, eccentricity, vertices, major and minor axes, focal chords, latera racta and directories). • to derive the standard forms of equations of an ellipse, find its elements and to draw the graphs of ellipses • to know that circle is a special case of an ellipse.
• Equation of an Ellipse with given elements Examples and Exercises.	• a) to find the equation of an ellipse with the following given elements: • major and minor axes, • two points, • foci, vertices or lengths of a latera recta faci, minor axis or length of a latus rectum.
• Tangents and Normals to an ellipse Examples and Exercises.	• To find a) the points of intersection of an ellipse with a line including the condition of tangency. b) the equation of a tangent in slope form, c) the equations of tangents and normals to an ellipse at point.
Hyperbola	
• Hyperbola and its elements Examples and Exercises.	• to know the concept of a hyperbola and its elements (centre, foci, eccentricity, focal chord, latera recta, directrices, transverse and conjugate axes) • to derive the standard forms of a equation of hyperbola, find its elements and draw the graphs.

• Equation of a hyperbola with given elements Examples and Exercises.	• to find the equation of a hyperbola with the following elements: - transverse and conjugate axes with centre at origin. - eccentricity, latera recta and transverse axis - focus, eccentricity and centre - focus, centre and directrix • To convert equation of a hyperbola to the standard form by translation of axes and be able to find the elements.
• Tangents and Normals to Hyperbola Examples and Exercises.	• To find a) the points of intersection of a hyperbola and a line including condition of tangency. b) the equation of a tangent in slope form. c) the equations of tangents and normals to a hyperbola at a point.
General equation of Conics	
• Translation and rotation of axes Examples and Exercises.	a) to know that the general form of the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a conic. Statement of the relevant theorem without proof. b) be able to find the conditions that general equation of 2nd degree represents a particular conic in the standard form when: 1. $a = b$, $h = 0$ (circle) 2. $a \neq b$ (both having same signs), $h = 0$ (ellipse) 3. $a \neq b$ (both having opposite signs), $h = 0$ (hyperbola) 4. $a = 0$ or $b = 0$ and $h = 0$ (parabola) c) be able to convert general equation of 2nd degree in the form of equation of a particular conic (circle, parabola, ellipse and hyperbola) in standard form by translation and rotation of axes and find their elements. d) be able to find equations of a tangent at a point to a conic represented by general equation of second degree. e) to know that the general equation of 2n ddegree represents a hyperbola from $a=b=g=f=0$ and $h\neq0$, $c\neq=0$
• Intersection of two Conics Examples and Exercises.	• Be able to know that two conics intersects in 1) four real points 2) two real points

	3) two coincident real points 4) one real point 5) no real point
UNIT VII VECTORS	
• Introduction of vector in a plane Examples and Exercises.	• To know a) definitions of scalar and vector quantities (and their notations); vector as an ordered pair of real numbers and as a directed line segment; position vector of a paint, magnitude of a vector, unit vector, negative of a vector, zero vector; equal vectors and parallel vectors. b) to add and subtract two vectors (triangle law of addition of two vectors); commutative and associative properties of addition of vectors; multiplication of a vector by a scalar; c) to find $\overrightarrow{AB}$ if position vectors of points A and B are given; ratio formula (position vector of the point which divides $\overrightarrow{AB}$ in a given ratio); position vector of mid-pint of a line segment (when position vectors of end points are given). d) application of vectors in proving problems of geometry.
• Introduction of Vector in Space. Examples and Exercises	• a) To know location of a point in space using Cartesian system; concept of vector in space; fundamental unit vectors (i.j.k) components of a vector $\bar{a}$ as $a_1\underline{i} + a_2\underline{J} + a_3\underline{k}$, magnitude of a vector, unit vector; parallel, collinear and coplanar vectors. b) To know direction angels, direction cosines and direction ratios of a vector; distance between two points; addition and subtraction of vectors and multiplication of a vector by a scalar in components form and their applications in geometry.
• Scalar Product of two vectors Examples and Exercises.	a) To know the definition of scalar (dot) product of two vectors i.e. $\bar{a}.\bar{b} = ab\cos\theta$ deducing the facts $\underline{i}.\underline{i} = 1,\ \underline{j}.\underline{j} = 1,\ \underline{k}.k = 1,\ \underline{i}.\underline{j} = 0,\ \underline{j}.\underline{k} = 0,$ $\underline{k}.\underline{i} = 0,\ \underline{a}.\underline{b} = \underline{b}.\underline{a}$ b) to know analytical expression of $\bar{a}.\bar{b}$

	i.e. if $\underline{a} = a_1\underline{i} + a_2\underline{j}\ i + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ *then* $\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ angle between two vectors; projection of one vector on another vector; properties of scalar product (parallel vectors, perpendicular vectors) c) Application of scalar product in solving problems of geometry and trigonometry; i.e. to prove that $\cos(\alpha \pm \beta) = \cos\alpha\ \cos\beta \pm \sin\alpha\ \sin\beta$ $a^2 = a^2 + b^2 - 2ab\cos\gamma$ $a = b\cos\gamma + \cos\beta$ *etc.*
• Vector Product of two Vectors Examples and Exercises.	• a) To know the definition of vector (cross) product of two vectors, i.e, $\underline{a} \times \underline{b} =$ $\lvert a\rvert\ \lvert b\rvert\ \sin\theta\ \hat{n}$ where θ is the measure of the angle between vectors $\underline{a}$ and $\underline{b}$ and $\hat{n}$ is the unit vector perpendicular to both $\underline{a}$ and $\underline{b}$; b) derivation of following results. $\underline{i} \times \underline{i} = 0$, $\underline{J} \times \underline{J} = o$, $\underline{k} \times \underline{k} = o$ $\underline{i} \times \underline{j} = \underline{k}$, $\underline{j} \times \underline{k} = \underline{i}$, $\underline{k} \times \underline{i} - \underline{J}$, $\underline{a} \times \underline{b} = -(\underline{b} \times \underline{a})$, $\underline{a} \times \underline{b} = 0$ c) To know deteminantal espression of the vector product of two vectors. d) To know properties of vector product of two vectors; (1) $\underline{a} \times \underline{b} = 0$ if and only if a is parallel to b (2) $\lvert\underline{a} \times \underline{b}\rvert$ = area of a parallelogram where a and b represent its adjacent sides. (3) 1/2 $\lvert\underline{a} \times \underline{b}\rvert$ =area of a triangle where a and b represent two sides. e) Application of cross product of vectors in trigonometry: i.e. to prove: $\sin(\alpha \pm \beta) = \sin\ \alpha\cos\beta \pm \cos\ \alpha\sin\beta$ $\dfrac{a}{\sin a} = \dfrac{b}{\sin B} = \dfrac{c}{\sin \gamma}$
• Scalar Triple Product of Vectors Examples and Exercises.	• a) To know definition of scalar triple product of vectors. $(\underline{a} \times \underline{b}).\underline{c}$,$(\underline{b} \times \underline{c}).\underline{a}$,$(\underline{c} \times \underline{a}).\ \underline{b}$. b) To know that $(\underline{a} \times \underline{b}).\underline{c} = \underline{a}.(\underline{b} \times \underline{c})$ i.e. dot and cross are inter-changeable. c) to know the determinental expression for scalar

	triple product. d) To find the volume of a parallelepiped and regular tetrahedron. e) Applications of vectors in solving simple problems of mechanics.

Instruction Objectives

(Expected Outcomes XI)

The students are expected to be able to

Unit I: Number Systems

i. Find out the roots of $x^2 + 1 = 0$ are not real but are imaginary and they are denoted by $\pm i$.

ii. Know that complex numbers and understand that the set of real number is a subset of complex numbers.

iii. Find the conjugate & modulus of a given complex number.

iv. Be able to represent a complex number in a Cartesian Plane i.e., $a+ib$ can be represented by the point (a, b).

v. Know that there is no ordering of complex numbers but equality can be defined.

vi. Find out the conjugate and modulus of given complex numbers.

vii. Interpret the modulus geometrically as the distance of the point from the origin.

viii. Perform the four fundamental operations in the set of complex numbers (both forms).

ix. Know and verify the following properties of complex number:

a. Commutative and associative properties of addition and multiplication.

b. Distributive property of multiplication over addition.

c. Additive and multiplicative identities.

d. Additive and multiplicative inverses.

e. The triangular in-equality (Geometric proof to be given).

x. Be able to find real and imaginary parts of $(x + iy)^n$ where $n = \pm 1, \pm 2, \pm 3$.

Unit II: Sets, Functions and Groups

i. Write the basic principles of logic.

ii. Give logical proofs of:

   a. Distributive property of union over intersection & intersection over union.

   b. Demorgan's Laws.

iii. Be able to solve problems pertaining to operations on sets.

iv. Prove the properties of operations on sets by using Venn Diagram.

v. Define a binary operation in a given set as function.

vi. Define the function as rule of correspondence.

vii. Define linear and quadratic functions.

viii. Define domain, co-domain and range.

ix. Define one to one and onto functions and inverse function.

x. Give illustrative examples of binary operations in a given set.

xi. Know the following properties of Binary operations:

a. closure

b. commutative (a binary operation may be non-commutative).

c. associative and non associative.

d. existence of identity and inverse with respect to a given binary operation.

xii. Know the definition of a group and be able to give illustrative examples of finite and infinite groups. (The examples should be taken from the sets of real numbers, complex numbers and matrices).

xiii. Know addition modulo and multiplication modulo and solve related problems.

xiv. Know and find the solution of $a * x = b$ in a group.

xv. Know and apply the laws:

a. cancellation laws.

$y * x = y * z \Rightarrow x = z$

$x * y = z * y \Rightarrow x = z$

b. $(x^{-1})^{-1} = x$ and $(xy)^{-1} = y^{-1} x^{-1}$

c. Prove the uniqueness of the identity element and inverse of each element of a group.

xvi. Define and distinguish between:

a. groupoid, a semigroup and a monoid.

b. finite and infinite groups.

c. commutative and non commutative groups.

UNIT – III Matrices and Determinants

i. Know the definition and meanings of an mxn matrix.

ii. Know the conditions under which two given matrices are equal.

iii. Find the sum, difference and product of two matrices upto order 4 x 4 with real entries & of matrices upto order 2x2 with complex entries.

iv. Know the definition of a determinant upto order 4 and be able to give illustrative examples.

v. Know the association between square matrix and determinant.

vi. Know and verify the following properties of matrices upto order 4 x 4.

a. commutative and associative properties of addition.

b. Multiplication is not, in general, commutative.

c. Additive and multiplicative identities.

vii. Know the properties of determinants and able to expand the given determinant upto order 4 x 4.

viii. Find the minors and cofactors of elements of a given determinant upto order 3.

ix. Write null and unit matrices upto order 4 x 4.

x. Know the definitions of singular and non-singular matrices.

xi. Find the transpose, adjoint and inverse of a given 3 x 3 matrix.

xii. Perform the elementary row and column operations on a matrix.

xiii. Can distinguish between the following types matrices. Upper and lower triangular, symmetric and anti-symmetric, Hermitian and anti-Hermitian, Echelon and reduced-Echelon forms.

xiv. Find inverse and rank of a matrix by applying row/column operations.

xv. Distinguish between a homogenous and non- homogenous linear equations in three variables by using matrices and Cramer's Rule.

Unit-IV Quadratic Equations

i. Can state the factor theorem of polynomial and its applications (Remainder Theorem) and synthetic division.

ii. Solve equations reducible to the quadratic form.

iii. Solve a system of two equations in two unknowns when:

a. One equation is quadratic and the other is linear.

b. Both equations are quadratic.

iv. Explain and discuss the nature of roots of a given quadratic equations.

v. Identity the relation between the roots and the coefficients of a quadratic equation.

vi. Form a quadratic equation when its roots are:

a. reciprocals/n times the roots of a given equation.

b. Increase/decrease by a given number the roots of a given equation.

vii. Find the cube and fourth roots of unity and know that $1+\omega+\omega^2=0$ and $\omega^3=1$

viii. Be able to solve problems involving quadratic equations.

Unit V Partial Fractions

i. Differentiate between proper, improper and rational expressions.

ii. Know that an improper rational expression is the sum of a polynomial and a proper rational expression.

iii. Split a proper rational expression into partial fractions when its denominator consists of:

   a. linear factors

   b. repeated linear factors upto 3

   c. irreducible quadratic factors

Unit VI Sequence and Series

i. Define the arithmetic, harmonic and geometric sequences and series.

ii. Find the general term of given arithmetic, harmonic and geometric sequences.

iii. Find the sum of first n terms of given arithmetic and geometric series.

iv. Find A, H & G and know that:

a. $A > G > H$

b. $AH = G^2$

Where A, H & G denote respectively the arithmetic, harmonic and geometric means between two positive real numbers.

v. Be able to insert "*n*" arithmetic, harmonic and geometric means between two given numbers.

vi. Be able to solve problems involving sequences and series.

vii. Know the meaning of $\sum n$, $\sum n^2$, $\sum n^3$ and their values.

viii. Find the sum of given series using then.

Unit VII Permutation, Combination and Probability

i. Identity the factorial notation and that $0! = 1$

ii. Define the permutation and be able to derive the formula for ^nP_r, when:

   a. All the things are different

   b. Some of the *n* things are alike (similar).

iii. Find arrangements of different things around a circle.

iv. Define the combination and be able to derive the formula for ^nC_r.

v. Prove that: $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

vi. Be able to solve problems involving permutations and combinations.

vii. Solve simple counting problems.

viii. Apply Venn diagram to find the probability of occurrence of any event.

ix. Know and apply the laws of additions & multiplication of probabilities.

x. Be able to solve problems related to above laws.

Unit VIII Mathematical Induction and Binomial Theorem

i. Know and verify the principle of Mathematical Induction and its simple applications.

ii. Be able to verity the formulas of:

   a. $\sum n$, $\sum n^2$ and $\sum n^3$

   c. sum of *n* terms of an AP and GP.

iii. Apply on conditional equations and in-equations.

iv. State and prove the binomial theorem for a positive integral index.

v. State the binomial theorem when the index is rational numbers.

vi. Apply the binomial theorem to find a particular term without expansion.

vii. Apply the theorem in computation.

viii. Apply the theorem in approximation and summation of series.

Unit IX Fundamentals of Trigonometry

i. Motivate the students to study trigonometry.

ii. Define a radian and radian function.

iii. Know to be able to identify radian as a unit of angular measure and be able to find the ralation between radian and degree measures of and angle.

iv. Know the relation between radian measure of a central angle, the radius and length of arc of a circle.

v. Define a general angle (i.e., including angles grater than 2π).

vi. Define angle in the standard position.

vii. Define and derive six basic trigonometric ratios.

viii. Find relations between the trigonometric ratios.

ix. Find the values of trigonometric rations.

x. Find sings of six basic trigonometric ratios in the four quadrants.

xi. Find the other five basic trigonometric ratios when one of them is given.

UNIT X Trigonometric Identities of sum and difference of angles.

i. Be able to prove fundamental low of trigonometry and its deductions.

ii. Know and prove the addition and subtraction formula of trigonometric ratios and all the deductions, there from including allied angles and half angles.

iii. Be able to apply these formulae to prove other trigonometric identities.

UNIT XI Trigonometric functions and their graphs

i. Find the period of trigonometric function.

ii. Know and find the domains and ranges of six basic trigonometric functions.

iii. Exhibit the six basic trigonometric functions graphically in the interval $[-2\pi, 2\pi]$.

iv. Know the fact that the graph of trigonometric functions are repeated for wider ranges.

UNIT XII Application of Trigonometry

i. Solve problems on heights and distance (involving right triangles).

ii. State and prove the laws of sines, cosines and tangents.

iii. Derive half angle formula in terms of sides.

iv. Solve oblique triangles.

v. Find the area of triangles in various eases.

vi. Find cirum-radius, in radius and radii of escribed circles in terms measures of the sides and angles of triangles.

vii. Solve problems involoving R, r, r_1, r_2, r_3, Δ.

UNIT XIII Inverse Trigonometric Functions

i. Know the definitions of domains and ranges of inverse trigonometric.

ii. Define the principal and general values of trigonometric functions.

iii. Establish and apply the formulas for inverse trigonometric functions.

UNIT XIV Solutions of Trigonometric Equations

i. Solve trigonometric equations.

ii. Find general solution of the equations.

iii. Discard the extraneous roots from the solution of trigonometric equations.

Instructional Objectives

(Expected Outcomes XII)

The students are expected to be able to

UNIT 1: Functions and Limits

i. Define the function as a binary relation.

ii. Find the domain and range of a given function.

iii. Define differentiate between following types of functions:

Algebraic, trigonometric, Inverse trigonometric, exponential, logarithmic, hyperbolic, inverse hyperbolic; explicit, parametric, even, odd and rational functions.

iv. Be able to draw the graphs of above mentioned functions.

v. Understand the concept of limit through the process of area of circle by inscribing polygons inside it.

vi. Understand the concept of limit by half of a Unit, and then half of the remainder and repeating the process indefinitely.

vii. Evaluate limits of various algebraic rational expressions.

viii. Define the theorems on addition, subtraction, multiplication and division of limits without proof and be able to solve problems involving these theorem.

ix. Evaluate $\underset{x \to 0}{Lim} \frac{\sin x}{x}$ and $\underset{x \to 0}{Lim} \left(1+\frac{1}{x}\right)^x$ and be able to apply them in the evaluation of limits of trigonometric and logarithmic functions.

x. Be able to draw graphs of:

a. explicit functions like $y = f(x)$, when $f(x) = e^y$, a^x, $\log_a x$, $\log e^x$

b. Implicit functions like $x^2 + y^2 = a^2; x^2/a^2 + y^2/b^2 = 1$

c. Parametric equations of functions like x = at2, y = 2at

$x = a \sec \theta, y = b \tan \theta$

d. Functions of the type

$$y = \begin{cases} x, when & 0 \le x \le 1 \\ x-1, when & 1 \le x \le 2 \end{cases} \quad and \quad \left. \begin{matrix} \dfrac{x^2-4}{x-2}, & x \ne 2 \\ x-2 & \end{matrix} \right\}$$

x. Differentiate between the meaning of continuity and discontinuity of function at a point and in an interval.

xi. Understand that continuous function has an ungroup graph and a discontinuous function has a broken graph.

UNIT II: Differentiation

i. Define and give geometrical interpretation of a derivative.

ii. Be able to find derivatives from first principle (simple eases only).

iii. Establish theorems derivatives of sum, difference, product and quotient of functions.

iv. Be able to differentiate algebraic, trigonometric, inverse trigonometric, logarithmic, exponential, hyperbolic and inverse hyperbolic functions and implicit functions.

v. Solve problem relating to velocity and acceleration.

vi. Apply chain rule in differentiating parametric functions and composite functions.

vii. Prove Maclaurin's and Taylor's theorems and their applications.

viii. Be able to calculate derivatives of second, third and fourth orders.

ix. Define increasing and decreasing functions.

x. Find maxima and minima and their simple problems.

xi. Bring as use derivative in solving problems of physical and biological sciences.

UNIT III: Integration

i. Define differentials.

ii. Be able to understand the process of finding the anti-derivatives is the inverse of the process of finding derivatives.

iii. Be able to find the anti-derivative of simple algebraic trigonometric and exponential functions.

iv. State and use theorems on integration.

v. Solve problems with the help of following techniques of integration:

a. By substitution

b. By partial fractions

c. By parts

vi. Define Definite Integral.

vii. Be able to find the area under a curve above x-axis and between two ordinates.

viii. Use anti-derivatives in solution of simple first order differential equations.

UNIT IV: Introduction to Analytic Geometry

i. Understand that a homogeneous equation of second degree in two variable is of the form: $ax^2 + 2hxy + by^2 = 0$ where a, b & $h, \in R$ and that it always represents a pair of straight line passing through the origin.

ii. He able to fine the angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$ and to derive the condition of perpendicular there from.

iii. Derive distance formula.

iv. Use the formula of division of line segment in a given ration internally & externally.

v. Be able to derive the different standard forms of the equations of a straight line.

vi. Prove that:

a. right bisectors of sides.

b. Bisectors of angles

c. Medians and altitudes of types.

vii. Define translation and rotation of axes.

viii. Transfer the linear equation $ax + by + c = 0$ in the standard forms of equations of straight line.

ix. Derive the distance of a point from a line.

x. Find area of a triangle in terms of coordinates of its vertices.

xi. Find the angle between two given lines in terms of their slopes and deduce the conditions of parallelism and perpendicular of two lines.

xii. Find the equation of straight line, parallel/perpendicular, to a given line.

xiii. Find the point of intersection of two given straight lines and the point of concurrency of three lines.

xiv. Express the equation of one, two three, straight lines in the matrix form.

xv. Derive that three lines given by : AX = 0 are concurrent, if A is a singular matrix.

UNIT V: Linear Inequalities and Linear Programming

i. Illustrate linear inequations in two variable graphically.

ii. Find region bonded by 2 or 3 simultaneous inequations of non-negative variables.

iii. Know and find feasible region and feasible solution.

iv. Have concept of linear programming.

v. Find optimal solution of linear objective function algebraically and graphically.

UNIT VI: Conic Sections

i Derive the following equations of circles.

a. $x^2 + y^2 = a^2$

b. $(x - a)^2 + (y - b)^2 = r^2$

and be able to find the centres and radii of circles with equations of the above form.

ii. Know that $x^2 + y^2 + 2gx + 2fy + c = 0$ represent circle with center at $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

iii. In general a straight line intersects a circle in tow points; the points may be real, distinct or coincident of both complex, and to identify the line intersecting in coincident points as tangent to the circle.

iv. Prove some important properties of circle (specially those of the secondary school stage) by analytical method.

v. Know that the two tangents drawn to a circle from a external point are equal in length.

vi. Be able to drive the standard equations of parabola, ellipse and hyperbola, namely:

   a. $y^2 = 4ax$ (or $x^2 = 4$ by) a, b, $\in$ R.

   b. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

   c. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

vii. Write the equations of the tangents and normals to conic sections at given point.

viii. Be able to know that a line intersects a conic section (equations given in the standard form), in general, in two points.

ix. Know that in general two conic sections intersect in four points.

x. Write the general form of equation.

   $ax^{-2} + 2hxy + by^2 + 2gx + 2\,fy + c = 0$

xi. Establish the following facts about general equation of conic:

   a. $a = b$, $h = 0$ (circle)

   b. $a \neq b$ (both having same signs), and $h = 0$ (Ellipse)

   c. $c \neq b$ (both having opposite signs), and $h = 0$ (hyperbola)

   d. $a = 0$ or $b = 0$ and $h = 0$ (parabola)

xii. Convert general form to the to the form of equation of a particular conic b translation and rotation of axes.

xiii. Apply that two conic intersect in:

   a. Four real points

   b. Two coincident real points.

   c. One real point.

   d. No real point.

   e.

Unit – VII

VECTORS

i) Know the quantities such as volume, time, change, mass, distance, energy potential have magnitude only and are called scalar quantities. Know that quantities such as displacement, force, acceleration, momentum have both magnitude and direction and are called quantities.

ii) Know about the frame of reference in rectangular Cartesian system for three dimensional space, coordinates of a point, the xy, yz and xz planes.

iii) Know the definition of magnitude, direction cosines and direction ration of a vector.

iv) Know the definition of unit vector.

v) Know to basic unit vectors:
   $\underline{i} = [1,0,0]$, $\underline{J} = [0,1,0]$, and $\underline{k} = [0,0,1]$

vi) Prove that the unit vector along a vector $a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ is

$$\frac{a_1 \quad i}{\sqrt{a_1^2 + a_2^2 + a_3^2}} + \frac{a2 \quad j}{\sqrt{a_1^2 + a_2^2 + a_3^2}} + \frac{a_3 \quad k}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

vii) Apply the position vector of a point w.r.t. the orgin.

viii) Apply the triangle and parallelogram laws of vectors.

ix) Be able to define the scalar product of vectors that is:
$\underline{a} \cdot \underline{b} = ab\cos\theta$ where θ is the angle between $\underline{a}$ and $\underline{b}$

x) Prove that:
a. $a \cdot \underline{b} = \underline{b} \cdot \underline{a}$
b. $\underline{i} \cdot \underline{i} = \underline{j} \cdot \underline{j} = \underline{k} \cdot \underline{k} = 1$ and $\underline{i} \cdot \underline{j} = \underline{j} \cdot \underline{k} = \underline{k} \cdot \underline{i} = 0$
c. If $a = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $b = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$
then $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$

xi) Prove scalar product obeys distributive laws.

xii) Prove $\underline{a} \cdot \underline{b} = 0$ if $\underline{a}$ and $\underline{b}$ are orthogonal.

xiii) Be able to define the vector product of vectors $\underline{a}$ and $\underline{b}$ as $\underline{a} \times \underline{b} = ab\sin\theta$ n to both $\underline{a}$ and $\underline{b}$ in the direction from $\underline{a}$ to $\underline{b}$.

xiv) Prove that:
a. $\underline{a} \times \underline{b} = -(\underline{b} \times \underline{a})$
b. $\underline{i} \times \underline{j} = \underline{k}$, $\underline{i} \times \underline{k} = \underline{i}$, $\underline{k} \times \underline{i} = \underline{j}$ and $\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$
c. $\underline{a} \times \underline{b} = 0$ if $\underline{a}$ is parallel to $\underline{b}$.
d. if $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$

$$\text{then } \underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

xv. Calculate scalar triple product $\underline{a}.(\underline{b} \times \underline{c})$ represents the volume of a parallelepiped.

xvi. Calculate $\underline{a} \cdot (\underline{b} \times \underline{c}) = \underline{b} \cdot (\underline{c} \times \underline{a}) = \underline{c} \cdot (\underline{a} \times \underline{b}) = c \cdot (\underline{a} \times \underline{b}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

xvii. Find volume of regular tetrahedron.

xviii. Apply vectors in proving some geometrical facts, trigonometric identities and sine cosine formulas.

xix. Apply vectors in solving simple problems of physics & engineering.

xx. Topic-Wise Weightage & Time (Class XI)

Unit	Topic	Weightage	Periods (40 minutes each)
I.	Numbers Systems	5%	07
II.	Sets Groups	10%	07 07
III.	Matrices Determinates	10%	07 07
IV.	Quadratic equations	8%	13
V.	Partial fractions	5%	07
VI.	Sequence & series	8%	13
VII.	Permutations and combination Probability	10%	07
VIII.	Mathematical induction Binomial theorem	12%	07 12
IX.	Fundamentals of trigonometry	5 %	07
X.	Trigonometric identities	7%	12
XI.	Trigonometric functions and graphs.	5%	07
XII.	Application of trigonometry	5%	07
XIII.	Inverse trigonometric functions	5%	07
XIV.	Trigonometric equations	5%	07
		100 %	150 (6 periods a week)

Pattern Of Question Paper

Type of questions	Marks	Choice
1) Objective type questions with negative marking (20) parts)	20	Nil
2) Direct application of Formulae/Theorems (5 Parts)	10	Nil
3) Comprehension (09 Parts out of 18)	50	Choice by giving alternative parts
4) Application of the Formulae/Theorems in complicated problems (3 Parts out of 6)	20	Choice by giving alternate parts

Note: The part of the question paper-containing question of the first two types covering the whole course should be printed separately and then answer sheets should be collected after half an hour. There should be at least four sets of objective type question paper containing the same questions in different orders.

Topic-Wise Weightage& Time (Class XI)

Unit	Topic		Weightage	Periods(40 minutes each)
I.	Functions & Limits.		6%	11
II.	Differentiation of functions	Polynomial & Algebraic. Trigonometry and inverse trigonometric Logarithmic & exponential.	15%	07 07 07
	Successive differentiation and theorems and Maximum/Minimum		5%	07
III.	Integration (Indefinite Integrals) Definite Integrals.	Standard forms By partial fractions By substitution By Parts Area under the curve Differential equations	29%	07 07 05 07 07 07
IV.	Introduction to analytic Geometry		8%	13
V.	Linear in-equalities Linear programming		7%	07 05
VI.	Conic sections	Circle Parabola Ellipse Hyperbola General equation	17%	12 04 04 04 04
VII.	Vectors	Introduction to vector & scalar product Vector product scalar triple product.	13%	07 07 04
			100%	150 (6 periods a week)

Pattern Of Question Paper

Type of questions	Marks	Choice
1) Objective type questions with negative marking (20) parts)	20	Nil
2) Direct application of Formulae/Theorem(5 Parts)	10	Nil
3) Comprehension (09 Parts out of 18)	50	Choice by giving alternative parts
4) Application of the Formulae/Theorems in complicated problems (3 Parts out of 6)	20	Choice by giving alternate parts

Note: The part of the question paper-containing question of the first two types covering the whole course should be printed separately and then answer sheets should be collected after half an hour. There should be at least four sets of objective type question paper containing the same questions in different orders.

Guide Lines for Textbook writers (higher Secondary School)

Textbook writing is an art. As all arts follow certain principles, so does art of writing textbooks. Undoubtedly, a person attempting to write a textbook does possess sufficient knowledge about the subject but it is the presentation of the subject matter that counts. The most important difficulty that an author has to face is to bring himself to the mental level of the students for whom he is writing the book. He has to arrange the subject matter in a psychologically logical manner. It is possible only if he knows the laws of learning and has full control over the language so as to use the vocabulary, which is in keeping with the vocabulary of the students.

Textbook writers in Mathematics for higher secondary schools are at least M.Sc. in the subject and have, undoubtedly, clear concepts of the terms and formulas they have to present. The first important point is the introduction of new topics so as to make the students curious to learn the topic. It is not proper to discuss the whole subject matter concerning a topic and then give only one exercise for practice. For example, the questions on the solution of quadratic equations consist of eleven different types. It should be divided into at least two parts, one of which should comprise radical equations in which it is necessary to check the answer so as to discard the extraneous roots.

While teaching binomial theorem for positive integral index, the author should write the whole series in one line at every step so that the addition of like terms is evident to the students. He should also instruct the students to write the proof of the theorem in the landscape form of their note books-which would help them to understand every step in writing the proof. The method of differentiation should be introduced with the help of elementary concepts of physics in which the definition of velocity and acceleration are used. The same is the ease in teaching definite integrals which are used in forming the equations of motion. The concepts of vectors, dot product and cross product should be taught keeping in view their application in mechanise.

A textbook writer must realize that he is to cater to the needs of the following four categories of the people:-

1. It is to be a teacher for private students.
2. It is to be a necessary tool for regular students.
3. it is to work as a guide for the inexperienced teachers.
4. It is to be helpful for the experienced teachers.

The author should make all possible efforts to correlate Mathematics with daily life and should include the questions on the application of Mathematics in other branches of knowledge.

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